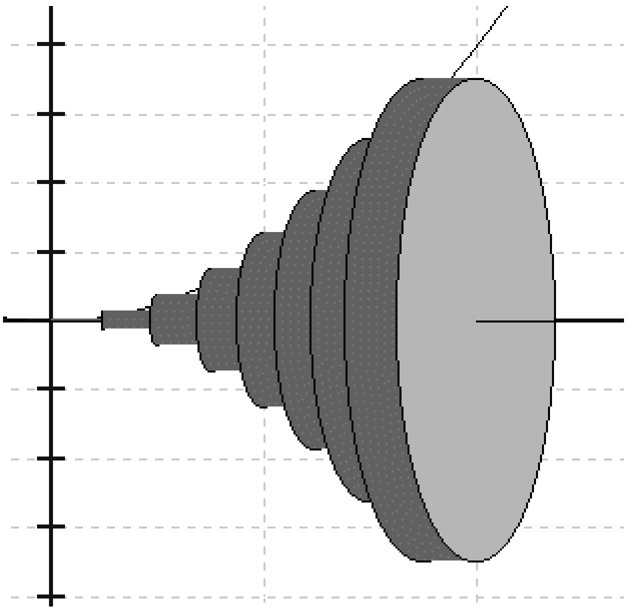
ABINGDON CONSORTIUM

FURTHER MATHEMATICS

Introduction to A level Further Maths



INDUCTION BOOKLET

SUMMER 2017

INTRODUCTION TO A LEVEL FURTHER MATHS AT ABINGDON CONSORTIUM

Thank you for choosing to study Further Mathematics in the sixth form at Abingdon Consortium. You are likely to sit modules in Core Pure Maths and either Statistics, Mechanics or Decision Maths at the end of year 12. The Mathematics Department is committed to ensuring that you make good progress throughout your A level or AS course. In order that you make the best possible start to the course, we have prepared this booklet.

It is <u>vitally important</u> that you spend some time working through the questions in this booklet over the summer - you will need to have a good knowledge of these topics <u>before</u> you commence your course in September. You will not necessarily have met all the topics before at GCSE so you may need to do some research before tackling the exercise – not necessarily every question, but enough to ensure you understand the topic thoroughly.

We will test you at the start of September to check how well you understand these topics, so it is important that you have looked at all the booklet before then.

We hope that you will use this introduction to give you a good start to your AS work and that it will help you enjoy and benefit from the course more.

Mr Whiting

Mr Moreton

Miss Twyford

Heads of Mathematics

Further Maths: Matrices

5 Add or subtract these matrices, as required. They are more difficult and you will need to take care with the negative signs.

a $\begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

b $\begin{pmatrix} 8 & -2 \\ 3 & 0 \end{pmatrix} - \begin{pmatrix} -6 & 4 \\ 5 & -2 \end{pmatrix}$

c $\begin{pmatrix} 5 \\ 0 \\ -3 \end{pmatrix} - \begin{pmatrix} 8 \\ -3 \\ 2 \end{pmatrix}$

d $\begin{pmatrix} 6 & 4 & -1 \end{pmatrix} + \begin{pmatrix} -2 & -1 & 5 \end{pmatrix}$

e $\begin{pmatrix} 6 & 0 \\ 1 & -4 \\ -2 & 7 \end{pmatrix} + \begin{pmatrix} 1 & 7 \\ 3 & 8 \\ -2 & -3 \end{pmatrix}$

f $\begin{pmatrix} 7 \\ 7 \\ 6 \end{pmatrix} + \begin{pmatrix} -5 \\ -8 \\ -9 \end{pmatrix}$

g $\begin{pmatrix} -2 & -3 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} 6 & 9 \\ -3 & -7 \end{pmatrix}$

h $\begin{pmatrix} 5 & 1 \\ 3 & 9 \\ -2 & -7 \end{pmatrix} + \begin{pmatrix} -4 & -4 \\ -7 & -6 \\ 5 & 3 \end{pmatrix}$

i $\begin{pmatrix} 6 & 2 \end{pmatrix} - \begin{pmatrix} -5 & 7 \end{pmatrix}$

j $\begin{pmatrix} -2 & -7 \end{pmatrix} - \begin{pmatrix} -8 & -3 \end{pmatrix}$

k $\begin{pmatrix} -1 \\ -6 \end{pmatrix} - \begin{pmatrix} -9 \\ -1 \end{pmatrix}$

l $\begin{pmatrix} 8 & 7 \\ -1 & -1 \end{pmatrix} + \begin{pmatrix} -2 & 4 \\ 6 & -1 \end{pmatrix}$

6 Find the value of each letter in these matrices.

a $\begin{pmatrix} 4 \\ a \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$

b $\begin{pmatrix} c \\ d \end{pmatrix} - \begin{pmatrix} 5 \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

c $\begin{pmatrix} e \\ f \\ 1 \end{pmatrix} + \begin{pmatrix} 8 \\ e \\ g \end{pmatrix} = \begin{pmatrix} 12 \\ 10 \\ f \end{pmatrix}$

d $\begin{pmatrix} h & i \\ 3 & 0 \end{pmatrix} + \begin{pmatrix} 4 & 5 \\ j & 3 \end{pmatrix} = \begin{pmatrix} 6 & 9 \\ i & k \end{pmatrix}$

e $\begin{pmatrix} 6 & 8 \end{pmatrix} - \begin{pmatrix} l & m \end{pmatrix} = \begin{pmatrix} 6 & l \end{pmatrix}$

f $\begin{pmatrix} n & 2 \\ p & r \end{pmatrix} - \begin{pmatrix} 4 & n \\ 1 & 8 \end{pmatrix} = \begin{pmatrix} 2 & p \\ q & -2 \end{pmatrix}$

Rather than write a matrix several times, it is quicker and easier to label it with a letter and write the letter instead.

1 $G = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}$ $H = \begin{pmatrix} 5 & 0 \\ 2 & 1 \end{pmatrix}$

Write the matrix
a $2G$ **b** $3H$
c $2G + 3H$.

2 $P = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}$ $Q = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$

Write the matrix
a $3P$ **b** $4Q$
c $3P + 4Q$.

3 $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \\ 4 & 5 \end{pmatrix}$ $B = \begin{pmatrix} 3 & 0 \\ 3 & 5 \\ 2 & 1 \end{pmatrix}$

Write the matrix
a $5A$ **b** $2B$
c $5A + 2B$.

4 $Y = \begin{pmatrix} 3 & 2 \\ 6 & 4 \end{pmatrix}$ $Z = \begin{pmatrix} 2 & 7 \\ 0 & 3 \end{pmatrix}$

Write the matrix
a $6Y$ **b** $3Z$
c $6Y + 3Z$.

5 $E = \begin{pmatrix} 3 & 4 & 2 \\ 5 & 6 & 3 \end{pmatrix}$ $F = \begin{pmatrix} 2 & 3 & 2 \\ 4 & 5 & 1 \end{pmatrix}$

Write the matrix
a $3E$ **b** $2F$
c $3E - 2F$.

6 $S = \begin{pmatrix} 4 \\ 5 \\ 7 \end{pmatrix}$ $T = \begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix}$

Write the matrix
a $4S$ **b** $3T$
c $4S - 3T$.

7 $I = \begin{pmatrix} 2 & 1 & 4 \\ 4 & 6 & 2 \\ 1 & 7 & 3 \end{pmatrix}$ $J = \begin{pmatrix} 3 & 2 & 2 \\ 4 & 1 & 3 \end{pmatrix}$ $K = \begin{pmatrix} 5 & 3 & 0 \\ 0 & 1 & 2 \end{pmatrix}$

$L = \begin{pmatrix} 0 & 1 & 2 \\ 2 & 3 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ $M = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$ $N = \begin{pmatrix} 4 & 5 \\ 2 & 1 \\ 0 & 2 \end{pmatrix}$

Use the lettered matrices above to perform these calculations.
If any are impossible, then say so.

a $I - L$ **b** $J + K$ **c** $N - M$
d $3J$ **e** $2K$ **f** $3J + 2K$
g $4M$ **h** $2N$ **i** $4M + 2N$
j $2I$ **k** $5M$ **l** $2I - 5M$
m $4J - K$ **n** $2I - L$ **o** $3K + 2N$

MECHANICS

6 In this question take g as $10\,\mathrm{m\,s^{-2}}$.

A small ball is released from rest. It falls for 2 seconds and is then brought to rest over the next 5 seconds. This motion is modelled in the speed-time graph Fig. 6.

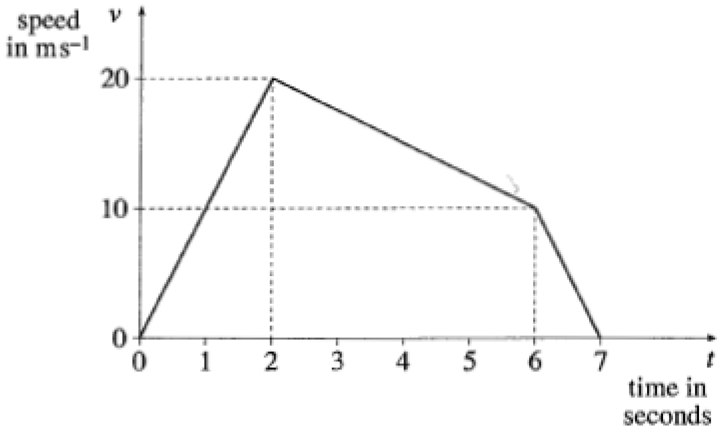


Fig. 6

For this model,

(i) calculate the distance fallen from $t = 0$ to $t = 7$, [3]

(ii) find the acceleration of the ball from $t = 2$ to $t = 6$, specifying the direction, [3]

(iii) obtain an expression in terms of t for the downward speed of the ball from $t = 2$ to $t = 6$, [3]

(iv) state the assumption that has been made about the resistance to motion from $t = 0$ to $t = 2$. [1]

The part of the motion from $t = 2$ to $t = 7$ is now modelled by $v = -\frac{3}{2}t^2 + \frac{19}{2}t + 7$.

(v) Verify that v agrees with the values given in Fig. 6 at $t = 2$, $t = 6$ and $t = 7$. [2]

(vi) Calculate the distance fallen from $t = 2$ to $t = 7$ according to this model. [7]

1 Fig. 1 shows four forces in equilibrium.

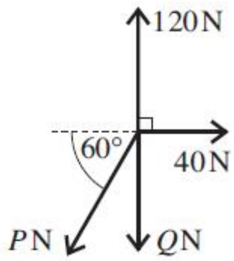


Fig. 1

(i) Find the value of P.

(ii) Hence find the value of Q.

1 Fig. 1.1 shows a circular cylinder of mass 100 kg being raised by a light, inextensible vertical wire AB. There is negligible air resistance.

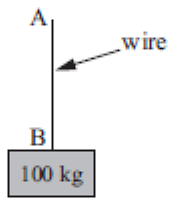


Fig. 1.1

(i) Calculate the acceleration of the cylinder when the tension in the wire is 1000 N. [3]

(ii) Calculate the tension in the wire when the cylinder has an upward acceleration of $0.8\,\text{m s}^{-2}$. [2]

The cylinder is now raised inside a fixed smooth vertical tube that prevents horizontal motion but provides negligible resistance to the upward motion of the cylinder. When the wire is inclined at 30° to the vertical, as shown in Fig. 1.2, the cylinder again has an upward acceleration of $0.8\,\text{m s}^{-2}$.

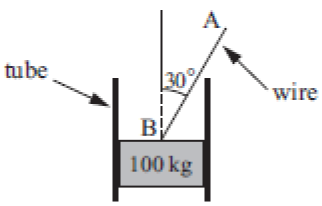


Fig. 1.2

(iii) Calculate the new tension in the wire. [3]

DECISION MATHS

3 An incomplete algorithm is specified in Fig. 3.

$f(x) = x^2 - 2$
Initial values: $L = 0, R = 2$.
Step 1 Compute $M = \dfrac{L+R}{2}$.
Step 2 Compute $f(M)$.
Step 3 If $f(M) < 0$ change the value of L to that of M.
Otherwise change the value of R to that of M.
Step 4 Go to Step 1.

Fig. 3

(i) Apply two iterations of the algorithm. [6]

(ii) After 10 iterations $L = 1.414063$, $R = 1.416016$, $M = 1.416016$ and $f(M) = 0.005100$.

Say what the algorithm achieves. [1]

(iii) Say what is needed to complete the algorithm. [1]

3 The following algorithm (J. M. Oudin, 1940) claims to compute the date of Easter Sunday in the Gregorian calendar system.
The algorithm uses the year, y, to give the month, m, and day, d, of Easter Sunday.
All variables are integers and **all remainders from division are dropped**. For example, 7 divided by 3 is 2 remainder 1. The remainder is dropped, giving the answer 2.

$c = y / 100$

$n = y - 19 \times (y / 19)$

$k = (c - 17) / 25$

$i = c - (c / 4) - (c - k) / 3 + (19 \times n) + 15$

$i = i - 30 \times (i / 30)$

$i = i - (i / 28) \times (1 - (i / 28) \times (29 / (i + 1)) \times ((21 - n) / 11))$

$j = y + (y / 4) + i + 2 - c + (c / 4)$

$j = j - 7 \times (j / 7)$

$p = i - j$

$m = 3 + (p + 40) / 44$

$d = p + 28 - 31 \times (m / 4)$

For example, for 2008:

$y = 2008$

$c = 2008/100 = 20$

$n = 2008 - 19 \times (2008/19) = 2008 - 19 \times (105) = 13$, etc.

Complete the calculation for 2008. [8]