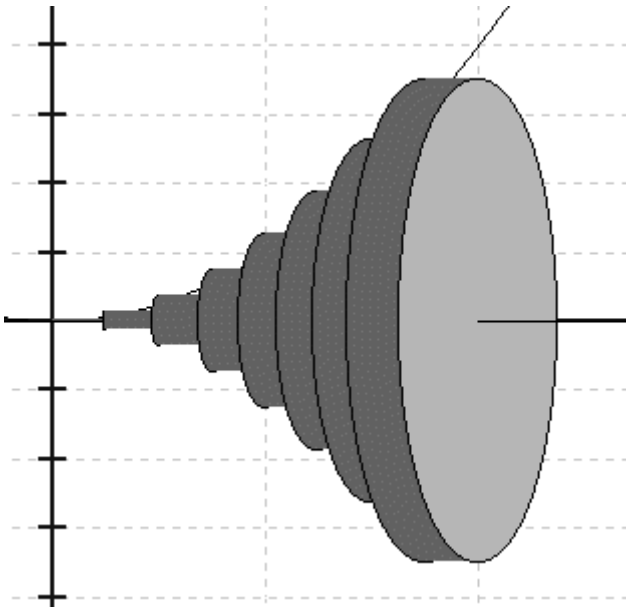
LARKMEAD SCHOOL

FURTHER MATHEMATICS

Introduction to A level Further Maths



INDUCTION BOOKLET

SUMMER 2019

INTRODUCTION TO A LEVEL FURTHER MATHS AT LARKMEAD SCHOOL

Thank you for choosing to study Further Mathematics in the sixth form at Larkmead School. You will work through modules in Core Pure Maths, Statistics and Mechanics. The Mathematics Faculty is committed to ensuring that you make good progress throughout your A level or AS course. In order that you make the best possible start to the course, we have prepared this booklet.

It is vitally important that you spend some time working through the questions in this booklet over the summer - you will need to have a good knowledge of these topics before you commence your course in September. You will not necessarily have met all the topics before at GCSE so you may need to do some research before tackling the exercise – not necessarily every question, but enough to ensure you understand the topic thoroughly.

We hope that you will use this introduction to give you a good start to your AS work and that it will help you enjoy and benefit from the course more.

Mrs Abbie Wilcock

Head of Mathematics

Further Maths: Matrices

5 Add or subtract these matrices, as required. They are more difficult and you will need to take care with the negative signs.

a $\begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

b $\begin{pmatrix} 8 & -2 \\ 3 & 0 \end{pmatrix} - \begin{pmatrix} -6 & 4 \\ 5 & -2 \end{pmatrix}$

c $\begin{pmatrix} 5 \\ 0 \\ -3 \end{pmatrix} - \begin{pmatrix} 8 \\ -3 \\ 2 \end{pmatrix}$

d $\begin{pmatrix} 6 & 4 & -1 \end{pmatrix} + \begin{pmatrix} -2 & -1 & 5 \end{pmatrix}$

e $\begin{pmatrix} 6 & 0 \\ 1 & -4 \\ -2 & 7 \end{pmatrix} + \begin{pmatrix} 1 & 7 \\ 3 & 8 \\ -2 & -3 \end{pmatrix}$

f $\begin{pmatrix} 7 \\ 7 \\ 6 \end{pmatrix} + \begin{pmatrix} -5 \\ -8 \\ -9 \end{pmatrix}$

g $\begin{pmatrix} -2 & -3 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} 6 & 9 \\ -3 & -7 \end{pmatrix}$

h $\begin{pmatrix} 5 & 1 \\ 3 & 9 \\ -2 & -7 \end{pmatrix} + \begin{pmatrix} -4 & -4 \\ -7 & -6 \\ 5 & 3 \end{pmatrix}$

i $\begin{pmatrix} 6 & 2 \end{pmatrix} - \begin{pmatrix} -5 & 7 \end{pmatrix}$

j $\begin{pmatrix} -2 & -7 \end{pmatrix} - \begin{pmatrix} -8 & -3 \end{pmatrix}$

k $\begin{pmatrix} -1 \\ -6 \end{pmatrix} - \begin{pmatrix} -9 \\ -1 \end{pmatrix}$

l $\begin{pmatrix} 8 & 7 \\ -1 & -1 \end{pmatrix} + \begin{pmatrix} -2 & 4 \\ 6 & -1 \end{pmatrix}$

6 Find the value of each letter in these matrices.

a $\begin{pmatrix} 4 \\ a \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$

b $\begin{pmatrix} c \\ d \end{pmatrix} - \begin{pmatrix} 5 \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

c $\begin{pmatrix} e \\ f \\ 1 \end{pmatrix} + \begin{pmatrix} 8 \\ e \\ g \end{pmatrix} = \begin{pmatrix} 12 \\ 10 \\ f \end{pmatrix}$

d $\begin{pmatrix} h & i \\ 3 & 0 \end{pmatrix} + \begin{pmatrix} 4 & 5 \\ j & 3 \end{pmatrix} = \begin{pmatrix} 6 & 9 \\ i & k \end{pmatrix}$

e $\begin{pmatrix} 6 & 8 \end{pmatrix} - \begin{pmatrix} l & m \end{pmatrix} = \begin{pmatrix} 6 & l \end{pmatrix}$

f $\begin{pmatrix} n & 2 \\ p & r \end{pmatrix} - \begin{pmatrix} 4 & n \\ 1 & 8 \end{pmatrix} = \begin{pmatrix} 2 & p \\ q & -2 \end{pmatrix}$

Rather than write a matrix several times, it is quicker and easier to label it with a letter and write the letter instead.

1 $G = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}$ $H = \begin{pmatrix} 5 & 0 \\ 2 & 1 \end{pmatrix}$

Write the matrix
a $2G$ b $3H$
c $2G + 3H$.

2 $P = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}$ $Q = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$

Write the matrix
a $3P$ b $4Q$
c $3P + 4Q$.

3 $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \\ 4 & 5 \end{pmatrix}$ $B = \begin{pmatrix} 3 & 0 \\ 3 & 5 \\ 2 & 1 \end{pmatrix}$

Write the matrix
a $5A$ b $2B$
c $5A + 2B$.

4 $Y = \begin{pmatrix} 3 & 2 \\ 6 & 4 \end{pmatrix}$ $Z = \begin{pmatrix} 2 & 7 \\ 0 & 3 \end{pmatrix}$

Write the matrix
a $6Y$ b $3Z$
c $6Y + 3Z$.

5 $E = \begin{pmatrix} 3 & 4 & 2 \\ 5 & 6 & 3 \end{pmatrix}$ $F = \begin{pmatrix} 2 & 3 & 2 \\ 4 & 5 & 1 \end{pmatrix}$

Write the matrix
a $3E$ b $2F$
c $3E - 2F$.

6 $S = \begin{pmatrix} 4 \\ 5 \\ 7 \end{pmatrix}$ $T = \begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix}$

Write the matrix
a $4S$ b $3T$
c $4S - 3T$.

7 $I = \begin{pmatrix} 2 & 1 & 4 \\ 4 & 6 & 2 \\ 1 & 7 & 3 \end{pmatrix}$ $J = \begin{pmatrix} 3 & 2 & 2 \\ 4 & 1 & 3 \end{pmatrix}$ $K = \begin{pmatrix} 5 & 3 & 0 \\ 0 & 1 & 2 \end{pmatrix}$

$L = \begin{pmatrix} 0 & 1 & 2 \\ 2 & 3 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ $M = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$ $N = \begin{pmatrix} 4 & 5 \\ 2 & 1 \\ 0 & 2 \end{pmatrix}$

Use the lettered matrices above to perform these calculations.
If any are impossible, then say so.

a $I - L$ b $J + K$ c $N - M$
d $3J$ e $2K$ f $3J + 2K$
g $4M$ h $2N$ i $4M + 2N$
j $2I$ k $5M$ l $2I - 5M$
m $4J - K$ n $2I - L$ o $3K + 2N$

MECHANICS

6 In this question take g as $10\,\text{m s}^{-2}$.

A small ball is released from rest. It falls for 2 seconds and is then brought to rest over the next 5 seconds. This motion is modelled in the speed-time graph Fig. 6.

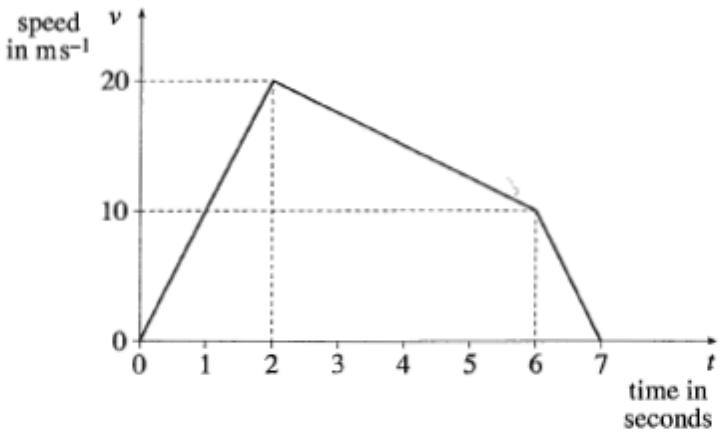


Fig. 6

For this model,

(i) calculate the distance fallen from $t = 0$ to $t = 7$, [3]

(ii) find the acceleration of the ball from $t = 2$ to $t = 6$, specifying the direction, [3]

(iii) obtain an expression in terms of t for the downward speed of the ball from $t = 2$ to $t = 6$, [3]

(iv) state the assumption that has been made about the resistance to motion from $t = 0$ to $t = 2$. [1]

The part of the motion from $t = 2$ to $t = 7$ is now modelled by $v = -\frac{3}{2}t^2 + \frac{19}{2}t + 7$.

(v) Verify that v agrees with the values given in Fig. 6 at $t = 2$, $t = 6$ and $t = 7$. [2]

(vi) Calculate the distance fallen from $t = 2$ to $t = 7$ according to this model. [7]

1 Fig. 1 shows four forces in equilibrium.

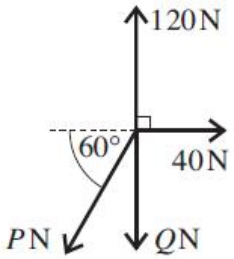


Fig. 1

(i) Find the value of P.

(ii) Hence find the value of Q.

[illegible] (50 marks)

1 Fig. 1.1 shows a circular cylinder of mass 100 kg being raised by a light, inextensible vertical wire AB. There is negligible air resistance.

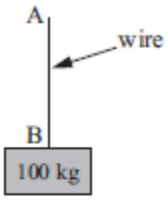


Fig. 1.1

(i) Calculate the acceleration of the cylinder when the tension in the wire is 1000 N. [3]

(ii) Calculate the tension in the wire when the cylinder has an upward acceleration of 0.8 m s^{-2}. [2]

The cylinder is now raised inside a fixed smooth vertical tube that prevents horizontal motion but provides negligible resistance to the upward motion of the cylinder. When the wire is inclined at 30° to the vertical, as shown in Fig. 1.2, the cylinder again has an upward acceleration of 0.8 m s^{-2}.

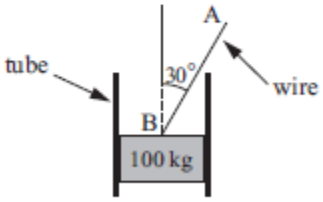


Fig. 1.2

(iii) Calculate the new tension in the wire. [3]